MATH 2050C Mathematical Analysis I
2019-20 Term 2
Problem Set 7
due on Mar 20, 2020 (Friday) at 11:59PM

Instructions: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. please do NOT come to campus to submit your completed assignments. Instead, you can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through CUHK Blackboard on/before the due date. Please remember to write down your name and student ID. You can refer to the webpage under "Useful Links" below about how to submit assignments through Blackboard. **No late homework will be accepted.** All the exercises below are taken from the textbook.

Required Readings: Chapter 3.4 (except the subsection on "Limit Superior and Limit Inferior")

Optional Readings: Limit Superior and Limit Inferior

Problems to hand in

Section 3.4: Exercise # 2, 4(b), 6, 9, 11

Suggested Exercises

Section 3.4: Exercise # 1, 3, 4(a), 5, 7, 8, 10, 12, 13, 14, 15, 16

Challenging Exercises (optional)

1. Section 3.4: Exercise # 17, 18, 19
2. Let (x_n) be the sequence of real numbers defined for $n \in \mathbb{N}$ by (with the convention that $0! = 1$)

$$x_n := \sum_{k=0}^{n} \frac{1}{k!} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}.$$

   (a) Prove that (x_n) converges to some real number $e \in \mathbb{R}$.

(b) Show that $e = \lim \left(1 + \frac{1}{n}\right)^n$.

(c) Prove that e is irrational.

3. This is a continuation of Challenging Exercise 2 of Problem Set 5.

(a) Find a sequence (x_n) of positive real numbers with $\limsup(x_n) = +\infty$ such that $\lim(s_n) = 0$.

(b) Let (y_n) be the sequence defined by $y_n := x_{n+1} - x_n$, $n \in \mathbb{N}$. Show that for $n \geq 2$,

$$x_n - s_n = \frac{1}{n} \sum_{k=1}^{n-1} k y_k.$$

Suppose that $\lim(ny_n) = 0$ and that (s_n) is convergent. Prove that (x_n) is convergent.